Solutions

10-701 Machine Learning
Fall 2023
Final Practice Problems

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1 CNN

1. **Select one:** What is the *primary* purpose of a convolutional layer in a CNN?

   ◯ To reduce the dimensions of the input image.

   ◯ To detect features such as edges and textures in the input image.

   ◯ To classify the input image into various categories.

   ◯ To flatten the input image across channels.

   B

2. Suppose you are building a CNN that takes input images with 3 channels, each of size 20×20 pixels. Your first convolutional layer takes the 3 input channels and produces 5 output channels; it uses 4×4 filters. It also uses a padding of 2 (along all sides of the image) and a stride of 2 (in both dimensions).

   i. **Numerical answer:** How many parameters are there in this convolutional layer, including the bias terms?

   The size of this convolutional layer can be represented as $5 \times 3 \times 4 \times 4$, giving 240 kernel parameters. We also have one bias term for each output channel for a total of 245 parameters.

   ii. **Numerical answer:** What is the dimensionality of the output of this convolutional layer? Include the channel dimension in your answer and make sure you clearly indicate which dimension is the channel dimension.

   Channel $\times$ Height $\times$ Width $= 5 \times 11 \times 11$

3. **Select one:** In CNNs, what is the *primary* advantage of using an inception block over a conventional convolutional layer?

   ◯ It allows for simultaneous use of filters of different sizes to capture features at various scales.

   ◯ It simplifies the network architecture by reducing the number of layers needed to learn the same set of features.

◯ It reduces the total number of parameters, making the network easier to train.

◯ It enhances the non-linearity of the network without increasing the depth.

A

2 Unsupervised Learning

1. For each of the **True or False** questions below, select the correct answer and briefly justify your selection in 1-2 concise sentences.

   i. For a fixed dataset and k, the k-means algorithm will always produce the same result if the initial centers are the same.

   ◯ True

   ◯ False

   True: the k-means algorithm is deterministic.

   ii. The k-means algorithm will always converge to the globally optimal solution.

   ◯ True

   ◯ False

   False: this depends on the initialization. Poor initializations can lead to local minima.

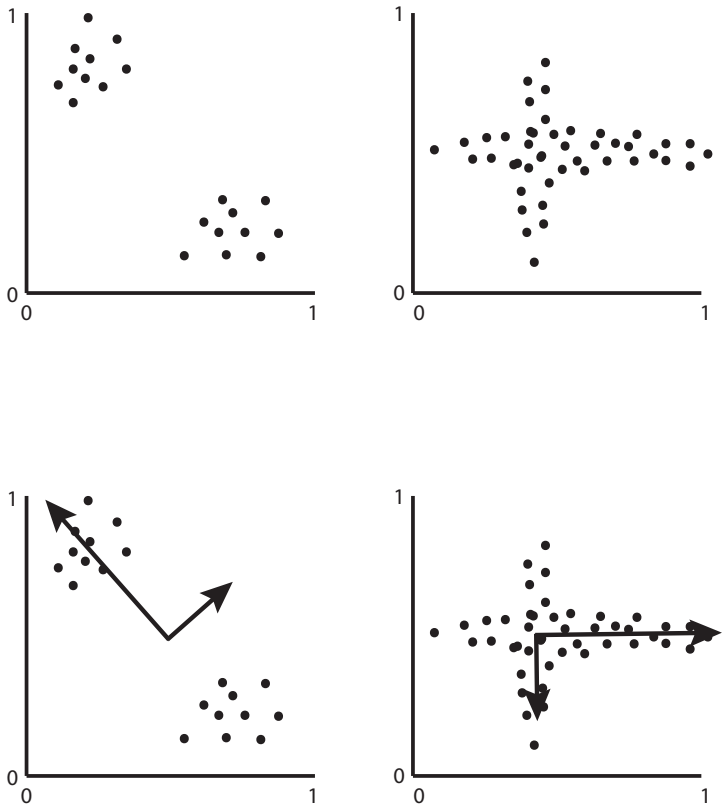
   iii. In the k-means algorithm, the objective function's value can increase, decrease or stay the same after each iteration.

   ◯ True

   ◯ False

False: the objective function's value can never increase from one iteration to the next.

2. Consider the following two datasets. On the figures below, draw arrows from the mean of the data to denote the direction and relative magnitudes of the principal components.



3. Given a dataset $\mathcal{D}$ consisting of N data points and D features, suppose you use PCA to project the dataset down to $d < D$ dimensions: let E be the squared reconstruction error of this projection.

   i. **Select one:** Now suppose that you add an extra data point to $\mathcal{D}$ so that $\mathcal{D}'$ consists of $N+1$ data points and D features. You once again use PCA to project $\mathcal{D}'$ to $d < D$ dimensions: let E' be the squared reconstruction error of this new projection. How do E and E' relate to one another?

   ◯ $E < E'$

   ◯ $E \leq E'$

   ◯ $E = E'$

   ◯ $E \geq E'$

   ◯ $E > E'$

   B

   ii. **Select one:** Now suppose that you use PCA to project the original dataset $\mathcal{D}$

down to $(d+1) < D$ dimensions instead of d dimensions: let E' be the squared reconstruction error of this new projection. How do E and E' relate to one another?

- ◯ $E < E'$
- ◯ $E \leq E'$
- ◯ $E = E'$
- ◯ $E \geq E'$
- ◯ $E > E'$

D

4. **Select all that apply:** Recall from lecture that autoencoders are trained by minimizing the reconstruction error between the inputs $\mathbf{x}$ and the corresponding outputs $\mathbf{x}'$. Given a dataset, $\mathcal{D} = \{\mathbf{x}^{(i)}\}_{i=1}^N$, which of the following are rational alternative objective functions for training autoencoders with backpropagation?

   - ☐ $\frac{1}{N}\sum_{i=1}^N \|\mathbf{x}^{(i)'}\|_2^2$
   - ☐ $\frac{1}{N}\sum_{i=1}^N \|\mathbf{x}^{(i)'} - \mathbf{x}^{(i)}\|_1$
   - ☐ $\max_i \|\mathbf{x}^{(i)'} - \mathbf{x}^{(i)}\|_2^2$
   - ☐ $\frac{1}{N}\sum_{i=1}^N \|\mathbf{x}^{(i)'} - \frac{1}{N}\sum_{i=1}^N \mathbf{x}^{(i)'}\|_2^2$
   - ☐ None of the above.

   C; B is not differentiable and thus, not a viable candidate for optimization via backpropagation. A and D are fundamentally different from the reconstruction error and will drive the autoencoder to learn meaningless representations. Note that C is a computationally expensive objective to optimize but is (potentially) feasible given the size of the dataset.

3 VAEs & GANs

1. **Math:** Suppose you have a black box which generates values from a normal distribution with mean 1 and variance 4 i.e., $x_i \sim \mathcal{N}(\mu = 1,\ \sigma^2 = 4)$. Using the *reparametrization trick* for VAEs, write down a formula for $y_i \sim \mathcal{N}(\mu = -1,\ \sigma^2 = 9)$ in terms of x_i.

   $y_i = 3 \cdot (\frac{x_i}{2} - 2)$

2. **Short answer:** Recall that the ELBO objective function used to train VAEs is short for "Evidence Lower BOund". In 2-3 concise sentences, briefly describe the quantity that the ELBO lower bounds and explain why it is used as the objective function.

   The ELBO lower bounds the evidence or the likelihood of the dataset. The evidence is intractable to optimize directly so instead VAEs are trained by optimizing the ELBO,

which is much easier to optimize. By optimizing a lower bound of the evidence, the likelihood of the dataset iteratively increases during training, making the ELBO a reasonable choice for the objective function.

3. **Select one:** Which of the following best describes the relationship between the discriminator and the generator in a GAN?

   ◯ Given an unlabelled dataset, the generator generates labels for the data points and the discriminator computes the likelihood of the generated labels.

   ◯ Given an unlabelled dataset, the discriminator clusters the data points and the generator generates new data points, conditioned on cluster assignments.

   ◯ Given an unlabelled dataset, the generator generates latent representations for each data point and the discriminator predicts whether some input vector is the latent representation for some data point from the dataset.

   ◯ Given an unlabelled dataset, the generator generates data points and the discriminator classifies data points as coming from the dataset of the generator.

   D

4. **Numerical answer:** Suppose that the probability of a some data point x is $p_{data}(x) = \frac{1}{10}$ and the probability of x under a generator G is $p_G(x) = \frac{1}{2}$. If D is the optimal discriminator in this setting, what is the probability that D assigns to x of being from the generator?

   $\frac{\frac{1}{2}}{\frac{1}{2}+\frac{1}{10}} = \frac{5}{6}$

5. **Short answer:** In a BEGAN, the binary discriminator is replaced with an autoencoder. In 1-2 concise sentences, describe the objective function that a BEGAN uses to train the discriminator.

   The discriminator in a BEGAN attempts to minimize the reconstruction error for data points from the real dataset and maximize the reconstruction error for data points generated by the generator.

4 RNN

1. **Select all that apply:** Which of the following statements is/are correct?

   - ☐ Training RNNs is difficult because of vanishing and/or exploding gradients.
   - ☐ Gradient clipping is an effective technique to address the vanishing gradient problem.
   - ☐ RNNs differ from feed-forward neural networks in that they have an additional weight matrix connecting hidden layers across time-steps.
   - ☐ RNNs can process sequences of arbitrary length, while feed-forward neural networks can not.
   - ☐ None of the above.

   A, C and D; gradient clipping is used to address exploding gradients

2. **Select one:** How do Long Short-Term Memory (LSTM) units address some of the limitations of standard RNNs?

   - ◯ LSTMs have lower computational costs, allowing for faster processing of sequences.
   - ◯ LSTMs simplify the structure of RNNs by decreasing the number of parameters, making them easier to train and less prone to overfitting.
   - ◯ LSTMs improve the feature extraction capabilities of RNNs.
   - ◯ LSTMs address the vanishing gradient problem through memory cells.

   D

3. **Short answer:** In 2-3 concise sentences, briefly describe the bi-directional RNN architecture and describe why they are *not* appropriate for language modelling.

   Bi-directional RNNs have hidden-layer connections between both the previous and the next time-steps. This means that bi-directional RNNs are incapable of performing next token prediction, a crucial task in language modelling.

5 Attention & Transformers

1. **Select all that apply:** For a fixed input-size and embedding dimension, which of the following statements is *not* true about multi-head attention relative to single-head attention?

   ◯ Multi-head attention is more suitable for parallel computation than single-head attention.

   ◯ Multi-head attention layers have more total parameters in their query, key and value matrices than single-head attention layers.

   ◯ Multi-head attention is compatible with more attention score functions than single-head attention.

   ◯ Multi-head attention are able to capture more diverse relationships between tokens than single-head attention.

   ◯ None of the above

   B and C

2. Suppose you have a transformer model that employs multi-head attention. The inputs to your model are sequences of T tokens and each token is represented by a d_M-dimensional embedding. Your model has H heads and for each head, the dimensionality of the key and query vectors is d_K and the dimensionality of the output vectors is d_V.

   i. **Math:** What are the dimensions of the key matrix *for one of the attention heads* i.e., K^h where $K^h = XW^h$?

   $T \times d_K$
   For a single attention head, the key matrix transforms the input embeddings into a new space defined by d_K. This transformation is applied separately for each token, resulting in a key matrix of dimension $T \times d_k$.

   ii. **Math:** What is the dimensionality of the multi-headed attention output *before any sort of concatenation*?

   $T \times d_V \times H$.
   For each head, the attention mechanism outputs a matrix of size $T \times d_V$; given H heads, before concatenating the results, we have a $T \times d_V \times H$-dimensional tensor as the output.

3. **Short answer:** In 2-3 concise sentences, briefly explain why positional encodings are used in transformer models.

In a standard transformer model, because each token attends to every other token, the output is order invariant. This can lead to suboptimal behavior as the relative position of tokens in a sequence can have an impact on their meanings. Positional encodings are used to include information about where in the sequence a given token is to address this shortcoming.

6 Pre-Training, Fine-Tuning and In-Context Learning

1. **True or False:** A model finetuned on a masked language modelling objective is appropriate for generative tasks such as text completion. Briefly justify your answer in 1-2 concise sentences

   ◯ True

   ◯ False

False, a masked language model predicts missing words and does not give a distribution over sequences, which is needed to perform generation.

2. **Short answer:** Give an example of chain-of-thought prompting. For full credit, your response must include the question you wish to be answered and cannot be one of the

examples provided in lecture.

Lots of potential examples, many correct answer will likely be in the form of word problems involving arithmetic.

3. **Select all that apply:** Which of the following techniques directly update the parameters of a pretrained language model?

   - ☐ Reinforcement learning from human feedback (RLHF)
   - ☐ Instruction finetuning
   - ☐ Few-shot learning
   - ☐ Soft-prompting
   - ☐ None of the above

   A and B

7 Robustness

1. **Select one:** What is the primary difference between targeted and untargeted adversarial attacks?

   ◯ Targeted attacks attempt to manipulate a model towards a specific class whereas untargeted attacks solely care about increasing model loss.

   ◯ Targeted attacks limit the magnitude of allowable perturbations whereas untargeted attacks allow for arbitrary perturbations.

   ◯ Targeted attacks can be optimized via gradient-based methods because the objective is differentiable whereas untargeted attacks cannot.

   ◯ Targeted attacks can be performed in multiclass classification settings whereas untargeted attacks can only be performed in binary classification settings.

   A

2. **Short answer:** In 2-3 concise sentences, briefly describe the primary difference between label shift and covariate shift; for full credit, your answer must address the assumptions made in both settings in terms of conditional probabilities.

   In label shift settings, we assume that the distribution over our labels is changing but the conditional distribution of the features given the labels remains constant whereas in covariate shift, we assume the distribution over possible inputs changes but the conditional distribution of labels given features remains constant.

3. **Proof:** Given a fixed classifier f, prove that the expected, column-normalized confusion matrices of f under two different label distributions p and q will be identical.

For some input x, let $f(x)$ be the model's prediction and let y^* be the true label of x. To show the desired result, it suffices to show that the probability that $f(x) = y_i$ and $y^* = y_j$ under the distribution p divided by the probability that $y^* = y_j$ under p doesn't change when the label distribution changes from p to q i.e.,

$$\frac{P(f(x) = y_i, y^* = y_j \mid \text{ label distribution } = p)}{P(y^* = y_j \mid \text{ label distribution } = p)} = \frac{P(f(x) = y_i, y^* = y_j \mid \text{ label distribution } = q)}{P(y^* = y_j \mid \text{ label distribution } = q)}$$

By Bayes rule,

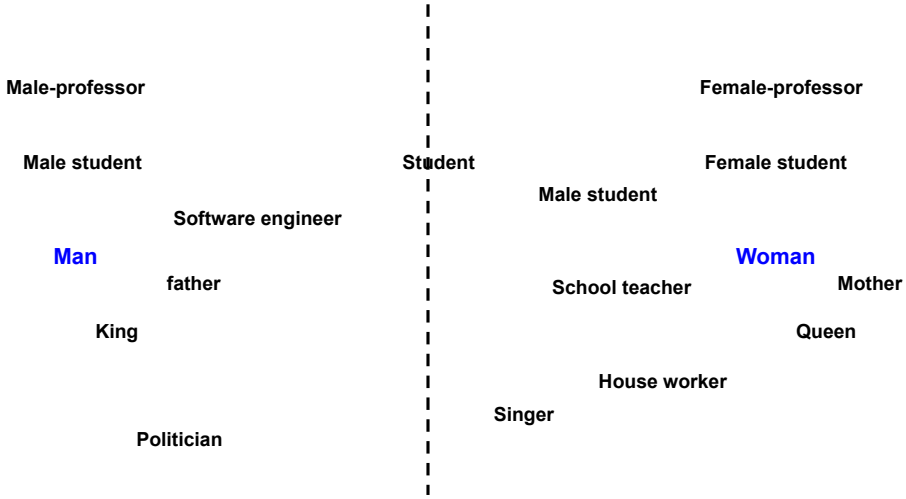
$$\begin{aligned}
&\frac{P(f(x) = y_i, y^* = y_j \mid \text{ label distribution } = p)}{P(y^* = y_j \mid \text{ label distribution } = p)} \\
&= \frac{P(f(x) = y_i \mid y^* = y_j, \text{ label distribution } = p) P(y^* = y_j \mid \text{ label distribution } = p)}{P(y^* = y_j \mid \text{ label distribution } = p)} \\
&= P(f(x) = y_i \mid y^* = y_j, \text{ label distribution } = p)
\end{aligned}$$

The distribution of $f(x)$ is conditionally independent of the label distribution given the true label y^* so we can conclude that

$$P(f(x) = y_i \mid y^* = y_j, \text{ label distribution } = p) = P(f(x) = y_i \mid y^* = y_j)$$

We can follow the exact same derivation using q as the label distribution and arrive at the same result ■

8 Bias



1. Consider the pre-trained word embedding space shown in the figure above.

   i. **Select all the apply:** Which of the following is a potential indicator that this word embedding is gender-biased in terms of occupation?

   - ☐ "man" - "king" = "woman" - "queen"
   - ☐ "man" - "software engineer" = "woman" - "school teacher"
   - ☐ "male professor" - "male student" = "female professor" - "female student"
   - ☐ "male professor" - "student" ≠ "female professor" - "student"
   - ☐ None of the above

   B

   ii. **Short answer:** Word embeddings like the one shown above are generally trained on web-scale text corpuses. In 1-2 concise sentences, briefly explain how gender bias, as defined by the spatial relationship between pairs of non-gendered words, is learned by word embeddings.

Since word embeddings are learned via (largely) human-generated text, it is likely that explicitly male nouns/pronouns appear more frequently in the context of certain professions (e.g., "computer programmer") while female nouns/pronouns appear more in the context of other professions e.g., "school teacher". Thus, in order to optimize it's objective function, the word-embedding assigns representations for these non-gendered terms that are closer in embedding space to one gender or another.

2. **Select one:** What is the relationship between demographic parity, separation and calibration in the general case?

   ◯ All three of these conditions can hold simultaneously.

   ◯ Any one of these conditions is not achievable if either of the other conditions is true.

   ◯ Any one of these conditions is not achievable if both of the other conditions are true.

   ◯ None of these conditions are achievable in the general case.

   C

9 Interpretability

1. **Short answer:** In 2-3 concise sentences, define an integrated gradient for a deep learning model and identify the primary issue with using an integrated gradient for explaining a model's predictions.

   An integrated gradient is the line integral of a model's gradient with respect to the parameters along a path from some "baseline" image and the input image. The primary issue with integrated gradients is the need to choose a baseline: different justifiable baselines can given wildly different line integrals.

2. **Select one:** Which of the following is the best description of a counterfactual explanation?

   ◯ Given an input that was predicted to be of one class, find the single feature that needs to be changed the least in order to change the model's prediction.

   ◯ Given an input that was predicted to be of one class, find the smallest set of features that if fixed, make it so that the model's prediction can never change regardless of the other features' values.

   ◯ Given an input that was predicted to be of one class, find the radius of the smallest ball centered at the input that includes a data point of another class.

◯ Given an input that was predicted to be of one class, find the nearest data point that would have been predicted to be of another class.

D